ANNUITY SAVINGS, NON-ANNUITY SAVINGS, HEALTH INVESTMENT AND BEQUESTS WITH OR WITHOUT PRIVATE INFORMATION

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SUMMARY

In this paper, we consider the optimal decisions of altruistic individuals on consumption, annuity savings, non-annuity, bequests and health investment when they are given any contract. We also examine the annuity return and quantity offered by firms in the presence of full-information and private information. We start from a simple case with exogenous survival rates, and then extend the model to the cases where survival rates are endogenous due to individuals' health investment. Four cases are further studied with endogenous survival rates—full-information, moral hazard, adverse selection and a mixture of moral hazard and adverse selection. We find that in a full-information case (the first best case), the amount of intentional bequests is equal to the accidental bequests. In a pure moral hazard case, the individuals' consumption path is strictly distorted; while in a pure adverse selection case, only those with low survival rates have their consumption paths distorted due to the externality of those with high survival rates. In the presence of both problems, the decisions of people with high survival rates are distorted the same way as in a pure moral hazard case while the decisions of people with low survival rates are further distorted due to the externality generated by those with high survival rates.

1. Introduction

In an influential paper, Yaari (1965) has shown that in the absence of bequest motives, individuals would fully annuitize savings to earn annuity returns that are higher than the market interest rates. Whereas in the presence of bequest motives, the uncertain lifetime would lead individuals to choose a "portfolio mix" not only to optimize consumption but to optimize savings and annuity purchases. However, bequest motives of annuity purchasers appear to have been rarely considered by the literature studying contract design of annuity firms. The individuals in our model are altruistic, who would value the bequest, both intentionally and accidentally left to their offspring.

In the paper, we distinguish the concepts of the "intentional bequests"---left to the offspring if the person survives to the second period, and the "accidental bequests" ---left to the offspring if the person cannot survive to the second period. This distinction fully captures the notion of "altruistic individuals", who care the child's wealth in both situations. While most literature focus only on one of the above scenarios, our paper examines this complication and its impacts on the consumer's optimal decision and the firm's profit maximization behaviors.

An important issue for a non-altruistic individual is how to allocate consumption, regular savings earning the market interest rate, bequests, and annuity purchases which would generate higher return than the market interest rate. If a contract provided is a combination of annuity quantity and return, the consumer would maximize his utility by choosing an optimal level of regular savings and bequests. At the same time, annuity firms would design contracts that are attractive enough for consumers and would earn non-negative profit.

However, the availability of information plays a key role in annuity firms' contract design. We would focus on the discrepancy of firms' contracts and individuals' decisions when individuals are given any contract. In the case of full information, when decisions of individuals can be observed, annuity firms would offer a utility-maximizing actuarially fair contract, which would be equivalent to consumers' utility maximization problem if they are given any contract. And the competitive equilibrium under full information is Pareto optimal. However, when individuals possess private information, such as a moral hazard and/or an adverse selection problem, the potential inefficiencies and/or negative externalities would be highlighted. Since individuals' decisions are not observable, and to overcome this information asymmetry, firms would design contracts that exclude the possibility of earning negative profit due to consumers' private information.

We examine the cases of full information, a pure moral hazard problem, a pure adverse selection problem and a mixture of moral hazard and adverse selection problem, and find a particular solution to each of these cases. We use a simplest case where the initial survival rate is exogenous to contrast with a full information case where health investment taken by individuals can be observed by annuity firms, and we find these two cases share several similar properties. In cases where private information presents, we use the first order condition constraint approach, proposed by Davies and Kuhn 1992, to deal with a moral hazard problem; and an incentive compatibility constraint is used to analyze an adverse selection problem.

We explore the role of bequests and regular (i.e. non-annuity) savings in consumers' utility maximization and in firms' contract design. We find that when annuity firms have full information on consumers, the consumer would leave the same amount of intentional and accidental bequests (regular savings). When there is private information, since consumer decisions on bequests and regular savings would not affect annuity firms' profit, the optimal conditions on savings and bequests are the same for the contract design.

We also investigate the role of health investment in consumers' utility maximization and firms' contract design. We show that there is a discrepancy between individuals' choice on health investment when given any contract and contractible health investment which guarantees non-negative profit of the firm. In the full-information case, consumers would take actions to affect their health state just as their annuity contracts settle. However, in the presence of private information, the utility would be lowered, and probably, health care would be overinvested, due to the implementation of policies aimed to overcome this information asymmetry.

Our contribution is to incorporate both "accidental bequests" and "intentional bequests", which fully captures the characteristics of the altruistic individuals. We characterize solutions of intertemporal consumption-saving decisions, bequests, health investment, and annuity purchases based on the availability of information. We differ from those in the literature in that we consider both consumers' optimal allocations given any contract and firms' behaviors when designing non-negative profit contracts. We fully discuss consumers' decisions and firms' contracts in four cases—full-information, a moral hazard problem, an adverse selection problem and a mixed situation with both moral hazard and adverse selection problems, while most of previous studies focus only on one or two aspects without taking planned and accidental bequests into consideration at the same time. We will argue that both types of bequests are important in the determination of not only total saving but also the division between annuity and non-annuity savings. We also discuss some special cases where bequest motivation is absent or survival rates are exogenous to gain a closer look into the role of information on welfare.

The rest of the paper proceeds as follows. Section 2 reviews previous literature. Section 3 describes the model. Section 4 presents a simplest case without health investment. Section 5 discusses health investment, information and contracts. Section 6 concludes the paper.

2. Literature Review

Yaari (1965) has established the fundamental theory of the consumer with uncertain lifetime. His paper discussed the role of the *Fisher-type utility function*— the normal form of expected utility representation and the *Marshall utility function*— the penalty function with direct preference on bequests. The *Marshall utility function* approach provides a rationale for including bequest motivation in the lifetime utility.

Based on the availability of the annuity, therefore, four cases have been discussed on the consumer's consumption-saving decision under uncertainty. Case A investigates the situation where the *Fisher utility function* is maximized subject to a wealth constraint when the insurance is unavailable. In this case, the consumer tends to discount the future more heavily. Case B considers the case where the *Marshall utility function* is maximized when the insurance is unavailable. The result shows that the consumer becomes more impatient if the marginal utility of consumption is greater than the marginal utility of bequests. Case C maximizes the *Fisher utility function* subject to the wealth constraint when the annuity is available. In this case, the consumer's assets (or liabilities) will always be held in the form of annuities due to a higher rate of return in annuity markets. Case D is actually a portfolio problem, since the altruistic consumer needs to optimize his purchase of annuities and the amount of savings that will be left for the offspring. The optimal saving plan and the optimal consumption plan are symmetric, which means when the insurance is available, the consumer can separate the consumption decision from the bequest decision.

The contribution of his paper is that it provides useful techniques—the *chance-constrained programming,* or the *Fisher-utility function* procedure and the *penalty function,* or the *Marshall utility function* procedure—for the analysis of uncertain lifetime of the consumer. It also provides the rationale for including bequests into the utility function, and derives the optimal consumption-saving plans when the insurance is or is not available. However, it does not consider consumers' decisions to change their survival chances.

Davies and Kuhn (1992) have proposed a simple model of annuities and social security when the hidden actions taken by the consumer could affect his longevity. Their paper has the following main results. First, in a pure moral hazard society, the mandatory actuarially fair social security system will never enhance the welfare due to the inability to "undo" the excessive public annuities and the fact that any increase in the level of the annuity from its optimal level will strictly reduce welfare. Second, a mandatory social security system with a moral hazard problem would have an ambiguous effect on longevity, rather than a usually expected positive effect. Third, in second-best annuity markets, social welfare can always be improved by a marginal longevity-reducing change in health behavior with actuarially fair annuities. Their paper analyzes the problem of competitive annuity firms, providing an insight for annuity firms' establishing contracts in a pure moral hazard economy. In doing so, it contrasts the optimal decisions of annuity purchasing and the welfare results under first best (full-information) context and second-best (private information) case, and examines the role of a mandatory social security system on welfare and longevity. One of the most important contributions of their paper is that it proposes a useful technique to deal with the contract in a moral hazard economy—the *First Order Condition constraint*, meaning that the competitive firms offering utility-maximizing actuarially -fair contracts should be subject to the constraint that the consumers will choose privately-optimal level of savings and health investment in response to any given contract.

However, their paper leaves several questions unanswered. First, indentifying three types of health-related goods, their paper assumes that the consumption of health-related goods directly affect the consumer's utility. The lack of consensus about the literature in the role of health expenditure also suggests that health care can be only considered as investment—affecting consumers' survival without inducing direct utility. Second, according to Yaari (1965), individuals with no bequest motivation would keep all the positive net assets in the form of annuities because annuities generate higher return than the market interest rate. In Davies and Kuhn's paper, regular savings is optimally chosen by the non-altruistic individual and can be positive. Third, as admitted by the authors, a complete analysis of social security system requires the consideration of both moral hazard and adverse selection problem.

Eckstein et al. (1985) described an economy with a pure adverse selection problem, where two groups of individuals keep their specific survival probability as private information when purchasing annuities from the markets. The presence of high survival rate individuals imposes a negative externality on other agents. In the case where the Rothschild-Stigliz equilibrium exists, such an externality is purely destructive in the sense that people with low survival rates are worse off than under a full-information context while those with high survival rates are not better off. In the case where the Wilson equilibrium exists, people with low survival rates are still worse off while people with high survival rates are better off. A mandatory social security program can always be welfare- enhancing in a pure adverse selection environment. Their work examines the conditions for a competitive equilibrium to exist in an adverse selection economy and the criteria to evaluate the desirability of government intervention. It also provides the economic intuition for an incentive constraint in annuity contracts. However, it is also possible that the survival rate is endogenous, rather than exogenously given when adverse selection is a problem.

Eichenbaum and Peled (1987) have investigated the existence of involuntary bequests when agents have no bequest motivation living in a pure adverse selection economy and agents' specific survival rates are private information. Their work is considered as an extension of Eckstein et al. (1985), in which no storable good is analyzed. They have established the results that the equilibrium in which the involuntary bequests are held by private agents cannot be Pareto optimal. A mandatory actuarially fair annuity program can result in the equilibrium without involuntary bequests that Pareto-dominates the initial equilibrium. Their paper contributes to the literature by showing that the involuntary bequests appear in equilibrium with private information even though agents have no bequest motivation. The inefficiency of the competitive equilibrium with involuntary bequests due to private information naturally induces the Pareto-improvement role of a mandatory social annuity plan. In line with Eckstein et al. (1985), they also show that a mandatory social annuity plan can be welfare-improving. However, a complete analysis of annuity markets still, requires consideration of both moral hazard and adverse selection problem, and endogenous survival depends on individuals' hidden actions.

Pauly (1974) has shown that in the presence of private information—moral hazard and adverse selection—the competitive outcome in insurance markets is non-optimal. It is proposed that public intervention may produce Pareto optimal improvements. His work underlines the analysis of both moral hazard and adverse selection problem in insurance markets, contributing to the literature of annuity markets. Actually, the techniques he used to analyze insurance companies can also be applied to the problem of annuity firms.

Platoni (2008) has established a particular model with annuity markets characterized by both moral hazard and adverse selection problems. The moral hazard problem arises as individuals choose the optimal level of health investment in responding to any given contract; while an adverse selection problem arises due to the heterogeneity in preference. In a pure moral hazard economy of his model, individuals with different types of preferences are worse off than in the full-information case in the sense that the Euler equations of both types of people are strictly distorted upwards and individuals tend to overinvestment in health care. In a pure adverse selection economy, the decisions of consumers with a stronger taste for old-age consumption and a greater joy of giving bequests are undistorted. By contrast, the decisions of consumers with a weaker taste for old-age consumption and a smaller joy of giving bequests are distorted in a way that they consume more in the first period and consume less in the second period. In the presence of both problems, a separating equilibrium is characterized by the fact that the welfare of more patient consumers is affected in the same way as in a pure moral hazard case while the welfare of less patient consumers is further distorted—a distortion coming from both moral hazard and adverse selection. In a pooling equilibrium, more patient consumers are better off than in a full-information case, while less patient consumers are worse off than in a full-information case.

The contributions of Platoni (2008) are as follows. First, it analyzes the cases of a pure moral hazard economy, an adverse selection problem, and the mixture of the two problems, and presents the main findings in different cases. Second, the paper is distinct from the previous literature in the way of inducing heterogeneity. In the previous studies of the annuity market, the heterogeneity across individuals is reflected in any given survival rates. In his paper, by endogenizing health investment in survival rate and the fact that time preferences affect health investment, the heterogeneity is derived from the difference in preference. This method is convenient to study a model with both moral hazard and adverse selection problems where individuals can choose an optimal level of health investment and have different types of survival rates. Third, it provides policy implications based on the result that government intervention may yield Pareto improvements under a separating equilibrium while the intervention may improve the well-being of individuals affected by the inefficiencies and negative externalities under a pooling equilibrium. However, it does not consider a case where the optimal level of annuities and non-annuity savings are both positive. In the real world, whether driven by precaution or joy of giving bequests, individuals tend to have a fraction of their total savings to be more liquid than annuity assets such as non-annuity savings as in most developed countries.

Zhang & Tang (2008) have explored the role of uninsurable medical expenses on the optimal decisions of annuitized savings and unannuitized savings. It also provides the interesting policy implications for government subsidies in preventative and remedial medical expenses for enhancing longevity. Their main findings are as followings. First, at a relatively lower initial survival rate, the consumer tends to fully annuitize his savings regardless of medical expenses, while at a relatively higher initial survival rate the consumer tends to have a positive non-annuitized savings, which increases as a further rise of the survival rate. Second, the paper illustrates the uniqueness of the solution for any given mortality and morbidity rates, which naturally induce the importance of comparative static analyses to see how the annuitized savings respond to the exogenous variables. Third, at a relatively high survival rate and a relatively low price of preventive health investment, the optimal level of preventive health care is positive, and government subsidies on remedial medical expenses would discourage preventive health investment.

One contribution of their paper is to provide the rationale for the positive unannuitized savings—the precautionary savings. The individuals in the model have an exogenous morbidity rate and they need to keep a fraction of total savings unannuitized in the sense that the unannuitized savings are more flexible to deal with the emergency. Meanwhile, the uniqueness of the solution contributes to the literature for the clarity in the relationship between longevity and annuitization. The third contribution of the paper is its policy implications. When the survival rate is high enough and the price of preventive health investment is low enough so that the optimal preventive health care is positive, the government should balance the subsidies on preventive health care and remedial health care since subsidies on preventive health investment may reduce future morbidity and remedial expenses.

There are several problems failed to be considered by their paper. In most developed countries, medical expenses are insurable by either public social security system or private insurance policy, or the mixture of the two. The paper considers only uninsurable or out-of-pocket medical expenses and ignores the full or partial insurable medical expenses, which may introduce different results from the paper. Another problem is when the morbidity rate in the second period depends on the health investment in the first period, a moral hazard problem arises if the consumer possesses private information. It is possible that, though the optimal health investment is positive, people may over-invest in health care, and thus lower the utility or welfare level. Therefore, it is necessary to include the analysis of annuity firms' behaviors under private information.

Pecchenino and Pollard (1997) have examined the effects of introducing actuarially fair annuity markets into an overlapping generation model of endogenous growth. They show that the full annuitization creates the maximized growth with a zero social security tax rate, while the full annuitization is not, in general optimal. The degree of annuitization that is dynamically optimal depends nonmonotonically on the expected length of retirement and the pay-as-you-go social security tax rate. Their work shed light on further studies of annuity markets in a dynamic context. It also provides policy implications for a government sponsored, actuarially fair pension system. However, there are limitations in this paper. One of them is the assumption of a percentage restriction on voluntary annuitization. This assumption is not a true reflection of real life in the sense that most countries do not set restricts on individuals' purchases of annuities. According to Yaari (1965), individuals without bequest motivation must fully annuitize their savings. It is expected that under both plans with positive nonannuitized savings, the choices are not optimal, and the growth rate is not maximized. Another problem is the inconsistency of assumptions. The paper assumes that if the non-altruistic individual dies when he is young at certain probability, his annuitized wealth is bequeathed to his child. This assumption naturally introduces a heterogeneity regarding to the bequest within individuals living in the same generation. At generation *T*, an individual's bequest should depend on not only his parent but the mortality history of the family (Zhang et al.(2003)). But in the analysis of bequest evolution, the authors simply assume that the bequests are equally allocated across all members of a generation.

Zhang et al. (2003) have analyzed the impact of a rising survival rate on economic growth in an overlapping generation model. In their model, the individuals in one generation are heterogeneous with respect to the unintentional bequest from the previous generations. They show that a decline in mortality can affect economic growth in a positive way due to the rise in the saving rate, and however, in a negative way due to the reduction in unintentional bequests. Starting from a high mortality rate, the net effect of a decline in mortality rate raises the growth rate, while staring from a low mortality rate, further reduction in mortality would lower the growth rate. Their work contributes to the literature in the following aspects. First, the findings are consistent with empirical evidence. Second, it provides useful techniques to deal with the evolution of accidental bequests in an overlapping generation model. Though individuals are heterogeneous within one generation, the aggregated savings and bequest can be traced, and thus, the capital accumulation can be characterized. However, since the paper has excluded the existence of annuity markets in the economy, it fails to discuss individuals' behaviors and growth when annuity markets are available. And the assumption of non-altruistic individuals leaving accidental bequests needs to be further considered.

Our paper extends the existing studies to analyze annuity savings, non-annuity savings, health investment and intergenerational transfers motivated by parental joy-of-giving. We will also divide this intergenerational transfer into planned and accidental portions and show both of them are important in the determination of not only the total amount of saving but also the division between annuity and non-annuity savings. We will introduce the basic features of the model in section 3. We will start in section 4 with a simple model whereby consumers have exogenous survival rates to enter old age. The model will be extended in section 5 to include health investment which may or may not be observable by insurance firms. A further extension will be made in section 5 to have different degrees of patience for old-age consumption and different degrees of the joy of giving bequests.

3. The Model

In this economy, there is a single non-storable good, an exogenously determined interest rate r and a large number of agents living for a maximum of two periods with a survival rate to the second period between 0 and 1. The mass of agents in the first stage of life is normalized to unity. Each agent in the first period is endowed with w units of good and receives a bequest b from the last generation. Agents allocate consumption intertemporally by purchasing annuities A in the first period which promises to pay him a higher rate of return α than the market interest rate r in the second period if the purchaser is still alive.

Agents are altruistic, motivated by joy of giving bequests. They each leave an intentional bequest b' to the next generation if they are alive in the second period or an accidental bequest $(1+r)s$ from their first period non-annuity or regular savings if they die before entering the second period. Both forms of bequests are valued in the agent's utility. This joint consideration of both types of bequests is a new feature compared to the literature, to the best of our knowledge.

The representative agent's expected utility is given by

$$U = u(C_1) + \beta\pi u(C_2) + \phi\pi u(b') + \phi(1-\pi)u[(1+r)s] \tag{1}$$

where $\pi \in (0,1)$ is the survival rate, $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'(x) \to \infty$ as $x \to 0$ and $u'(x) \to 0$ as $x \to \infty$. $\beta, \phi \in (0,1)$ are the discount factors and the assumption $\phi < \beta$ indicates agents value more of their own consumption in the second period than bequests left for the next generation. Our analysis will proceed from simple to more complex cases in the rest of this paper in order to better understand the forces at work.

4. The Simplest Case: An Exogenous Survival Rate

In this case, agents live through to the end of the second period with an exogenous survival rate of $p \in (0,1)$. In the annuity market, “the actuarial fairness condition” holds for market clearing; that is, the expected value of interest payments on the annuity is equal to the market interest rate.

$$p(1+\alpha) = 1 + r \tag{2}$$

For analytical tractability, we adopt a popularly used logarithmic utility function. The representative agent’s program is given by

$$Max_{A,s,b'} U = \ln C_1 + \beta p \ln C_2 + \phi p \ln b' + \phi(1-p)\ln[(1+r)s]$$

$$s.t. \qquad C_1 = w + b - A - s$$

$$C_2 = (1+\alpha)A + (1+r)s - b'$$

With market clearing condition (2), we have the following first order conditions, which define a competitive equilibrium with an exogenous survival rate.

$$(A) \qquad \frac{1}{C_1} = \frac{\beta(1+r)}{C_2} \tag{3}$$

$$(s) \qquad \frac{1}{C_1} = \frac{p\beta(1+r)}{C_2} + \frac{(1-p)\phi}{s} \tag{4}$$

$$(b') \qquad \frac{\beta}{C_2} = \frac{\phi}{b'} \tag{5}$$

Equation (3) is the optimal condition on annuity purchase, which can be interpreted as the Euler equation for consumption in both periods. Equation (4) is the optimal condition on non-annuity savings, including a new component in the marginal benefit of non-annuity saving derived from accidental bequests compared to the literature. Equation (5) is the optimal condition governing planned bequests.

Proposition 1: *With an exogenous survival rate, the representative agent allocates a constant ratio of annuity purchases to total savings, which is increasing in the survival rate ,and decreasing in the bequest motivation.*

Proof. Equation (5) and the budget constraint of C_2 imply

$$C_2 = \frac{\beta(1+r)}{\beta+\phi}\left(\frac{A}{p}+s\right) \tag{6}$$

Equations (6) and (3) give

$$(\beta+\phi)C_1 = \frac{A}{p}+s \tag{7}$$

Equations (3) and (4) imply

$$C_1 = \frac{s}{\phi} \tag{8}$$

Equations (7) and (8) give

$$\frac{A}{s} = \frac{p\beta}{\phi} \tag{9}$$

If we consider the total savings as $A+s$, then define $\gamma = \frac{A}{A+s}$ as the proportion of annuity purchases to total savings,

$$\gamma = \frac{p\beta}{p\beta + \phi} \tag{10}$$

Equation (10) states that each agent allocates γ of his total savings to annuity purchase and $(1-\gamma)$ to accidental bequests to a future generation. As p or (and) β increases, the ratio of annuity purchases to total savings increases, and as ϕ increases this ratio decreases. Q.E.D.

The intuition behind is that people are willing to spend more of their savings on annuity purchases as they expect to live longer or (and) have weaker bequest motives. In an extreme case where $\phi = 0$,we have $\gamma = 1$; that is, the representative agent allocates all the savings to annuity purchases. This coincides with Yaari (1965) that the consumer with no bequest motive will always hold his assets in annuity form rather than regular savings.

As in the literature, savings, annuities and young-age consumption are proportional to the total income in our model, which can be seen from equations (8), (9) and the budget constraint of C_1,

$$s = \frac{\phi}{1 + p\beta + \phi}(w+b) \tag{11}$$

$$A = \frac{p\beta}{1 + p\beta + \phi}(w+b) \tag{12}$$

$$C_1 = \frac{1}{1+p\beta+\phi}(w+b) \tag{13}$$

However, it is interesting to obtain a new relationship between the amounts of planned and accidental bequests:

Proposition 2: *The amounts of planned and accidental bequests are equal.*

Proof. From equation (5), we have

$$b' = \frac{\phi C_2}{\beta} \tag{14}$$

Equations (3), (8) and (14) give

$b' = (1+r)s$. Q.E.D.

The result in Proposition 2 is new compared to the literature and can be very helpful for the understanding of intergenerational transfers when allowing for a realistic annuity market. According to Kotlikoff and Summers (1981), bequests account for nearly half of capital accumulation in the United States. However, there is no joint consideration about both planned and accidental bequests despite little doubt in the literature about their co-existence in the real world. For example, some studies assume away bequests altogether and tread the saving of savors who die before entering old age as waste; see, e.g. Ehrlich and Lui (1991). Some studies ignore accidental bequests and assume that annuity saving is the only form of saving, implying that bequests are independent of one's family mortality history; see, e.g., Zhang et al. (2001). Some studies pay attention to non-annuity saving, such as precautionary saving, and assume away annuity markets entirely, leading to the dependence of bequests on one's family mortality history; see Abel (1985), Huggett (1996) and Zhang et al. (2003). Our results in Proposition 2 reconcile these different approaches and provide a useful and simple way to track down the wealth distribution over different generations. In our model, as bequests are the same whether one survives to the end of lifetime, the total amount of bequests one receives is independent of the family mortality history.

5. Health Investment, Information and Annuity Contracts

In this section, we modify the simplest case by adding two assumptions. First, we assume that the survival rate can be improved through health investment. Agents survive to the second period with probability $\pi(h) \in (0,1)$, where h represents the investment on health care. $h \in [0, \bar{h}]$, where $\bar{h} = w + b$. We specify the function π as

$$\pi(h) = 1 - \frac{a}{e^h} \ , \quad 0 < a < 1 \tag{15}$$

$\pi \to \bar{\pi}$ as $h \to \bar{h}$ and $\pi \to \underline{\pi}$ as $h \to 0$. $\bar{\pi} = 1 - \frac{a}{e^{\bar{h}}}$ is the upper bound of survival rate by investing all the income in health care, $\underline{\pi} = 1 - a$ is the natural survival rate with no health investment, $0 < \underline{\pi} \le \pi(h) \le \bar{\pi} < 1$.

$$\pi'(h) = ae^{-h} > 0 \ , \ \pi''(h) = -ae^{-h} < 0 \ .$$

Second, different from the simplest case where we implicitly assume consumers can choose the quantity of annuity purchases *A*, here we assume each contract provided by annuity firms is a combination of quantity and return on annuities[1], a contract (A, α). (see, e.g., Eckstein et al.1985,Davies & Kuhn 1992,Platoni 2007.)

[1] Rothchild and Stigliz argued that price competition is a special case of price-quantity competition. Most literature on annuity firms’ behavior assumes the contract offered by firms is a quantity-return combination.

5.1. The Consumer's Problem

Given a contract (A, α), the consumer maximizes utility by allocating consumption plan and making decisions on savings, bequests and health investment (s, b', h).

The problem of a representative agent with health investment is given by

$$Max_{s,b',h} U = \ln C_1 + \beta\pi(h)\ln C_2 + \phi\pi(h)\ln b' + \phi[1-\pi(h)]\ln[(1+r)s]$$

$$s.t. \qquad C_1 = w + b - A - s - h$$

$$C_2 = (1+\alpha)A + (1+r)s - b'$$

The first order conditions are

$$(s) \qquad \frac{1}{C_1} = \frac{\beta\pi(h)(1+r)}{C_2} + \frac{[1-\pi(h)]\phi}{s} \tag{16}$$

$$(b') \qquad \frac{\beta}{C_2} = \frac{\phi}{b'} \tag{17}$$

$$(h) \qquad \frac{1}{C_1} = \pi'\beta \ln C_2 + \pi'\phi \ln b' - \pi'\phi \ln[(1+r)s] \tag{18}$$

Equations (16)-(18) give the consumer's optimal decisions of savings, bequests and health investment when given any annuity contract. Equation (16) is the optimal condition on non-annuity savings, which is similar to equation (4). Equation (17) is the optimal condition governing planned bequests similar to equation (5). Equation (18) is new, presenting the optimal decision on health investment. It can be seen from equations

(16)-(18), the optimal decisions on savings, planned bequests and health investment can be expressed by (A,α). From equation (17) and budget constraints, we have

$$C_2 = \frac{\beta}{\beta+\phi}[(1+\alpha)A+(1+r)s] \tag{19}$$

$$b' = \frac{\phi}{\beta+\phi}[(1+\alpha)A+(1+r)s] \tag{20}$$

The relation between h and (A,α) are implicitly given by (18). The remaining analyses of the problems of firms differ among cases with or without private information on consumers' health investment.

5.2. The Firm's Problem

Generally, perfect competitive annuity firms maximize profit by designing the contract with both annuity quantity A and annuity return α, and take consumers' behavior into account. However, from the consumer's problem, we know that given any contract (A,α), the consumers would accordingly choose (s,b',h) to maximize the utility. Among all the contracts, the consumer would only purchase the contract that generates the maximum utility; that is, the contract (A,α) offered by the firm has to maximize consumer's indirect utility $U(A,\alpha)$, i.e.,

$$A,\alpha \in \arg Max_{A,\alpha} U(s(A,\alpha), b'(A,\alpha), h(A,\alpha))$$

Therefore, we can actually transform the firm's problem into another one, rather than the usually-adopted profit maximization problem. The firm knows that the consumer would only purchase the contract that gives her the maximum indirect utility, and this contract has to earn non-negative profit from the firm's concern. Thus, the general form of the firm's problem in our paper is defined by

$$Max_{A,\alpha} U(A,\alpha)$$

$$s.t. \quad 1+r-(1+\alpha)\pi(h) \geq 0$$

The inequality constraint guarantees that each indirect-utility-maximizing contract earns non-negative profit. This transformation is commonly used in the literature about annuity firms' behavior.[2]

5.2.1 Full-information Private Annuities

Annuity firms care about the individual decision on health investment which would affect the firms' profit via the non-negative-profit constraint, but firms do not care other decisions made by individuals, such as savings and bequests simply because these variables do not affect the firm's profit. With full information, the annuity firm can observe health investment taken by individuals, and health investment is contractible. Since the firm knows that individuals would take actions to improve health and thus affect annuity returns, and these actions and improvement in health may generate

[2] E.g. Pauly 1974,Eckstein et al 1985, Echienbaum and Peled 1987, Davies and Kuhn 1985,Platoni 2008,Eichenbaum and Peled 1987.

negative profit to the firm, it has to design a contract (A,α) and a level of h that maximize the consumer's utility and guarantee non-negative profit of the firm.

$$Max_{A,\alpha,h} U(A,\alpha) = \ln(w+b-A-s-h) + \beta\pi(h)\ln\left[(1+\alpha)A+(1+r)s-b'\right] +$$
$$\phi\pi(h)\ln b' + \phi[1-\pi(h)]\ln[(1+r)s]$$

$$s.t. \quad 1+r \geq (1+\alpha)\pi(h)$$

where s, b' given by equations (16) and (17).

Note that the inequality constraint is non-binding. Thus, we will consider the case where the firm earns zero profit; that is, the inequality is substituted with the equality, a linear relation between α and $\pi(h)$. By "the envelope theorem", the first order conditions are

$$(A) \qquad \frac{1}{C_1} = \frac{\beta(1+r)}{C_2} \tag{3a}$$

$$(h) \qquad \frac{1}{C_1} = \pi'\beta\ln C_2 - \frac{\beta(1+r)A\pi'}{\pi(h)C_2} + \pi'\phi\ln b' - \pi'\phi\ln[(1+r)s] \tag{21}$$

The solution to the competitive equilibrium with full-information is given by equation (3a) and (21) from the firm's side , and (16) and (17) from the consumer's side. Under full information we have several similar results as derived from the simplest case.

$$\text{(I)} \qquad \frac{A}{s} = \frac{\pi(h)\beta}{\phi} \ .$$

$$\text{(II)} \qquad s = \frac{\phi}{1+\pi(h)\beta+\phi}(w+b-h)$$

$$A = \frac{\pi(h)\beta}{1+\pi(h)\beta+\phi}(w+b-h)$$

$$C_1 = \frac{1}{1+\pi(h)\beta+\phi}(w+b-h)$$

(III) $\quad b' = (1+r)s$

Annuity purchase, consumption and savings are proportional to total income deducted by health investment. And the amounts of planned and accidental bequests are equal. These results and equation (21) give a "health investment rule" -- health investment can be determined by

$$\frac{1+\pi(h)\beta+\varphi}{\pi'\beta(w+b-h)} + \ln[1+\pi(h)\beta+\phi] = \ln[\beta(1+r)(w+b-h)] - 1 \qquad (22)$$

Proposition 3: *Under full-information, individual investment in health is increasing in total income, and decreasing in bequest motive, if the total income w+b is large enough; that is*

$$\frac{\partial h}{\partial(w+b)} > 0, \quad \frac{\partial h}{\partial \phi} < 0$$

Proof.

From equation (22), using the implicit function theorem, we have

$$\frac{\partial h}{\partial(w+b-h)}=\frac{1+\pi(h)\beta+\phi}{(w+b-h)^2}+\frac{\pi'\beta}{w+b-h}\bigg/\pi'\beta\left[\frac{2}{w+b-h}+\frac{\pi'\beta}{1+\pi(h)\beta+\phi}+\frac{1+\pi(h)\beta+\phi}{\pi'\beta(w+b-h)^2}+\ln\left[\frac{\beta(1+r)(w+b)}{1+\pi(h)\beta+\phi}\right]-1\right]$$

$$\frac{\partial h}{\partial\phi}=-\left[\frac{1}{w+b-h}+\frac{\pi'\beta}{1+\pi(h)\beta+\phi}\right]\bigg/\pi'\beta\left[\frac{2}{w+b-h}+\frac{\pi'\beta}{1+\pi(h)\beta+\phi}+\frac{1+\pi(h)\beta+\phi}{\pi'\beta(w+b-h)^2}+\ln\left[\frac{\beta(1+r)(w+b)}{1+\pi(h)\beta+\phi}\right]-1\right]$$

Since $0\le h\le w+b, 0<\beta<1$, and if

$$\frac{2}{w+b-h}+\frac{\pi'\beta}{1+\pi(h)\beta+\phi}+\frac{1+\pi(h)\beta+\phi}{\pi'\beta(w+b-h)^2}+\ln\left[\frac{\beta(1+r)(w+b)}{1+\pi(h)\beta+\phi}\right]>1\ ,$$

We have $\frac{\partial h}{\partial(w+b)}>0$, $\frac{\partial h}{\partial\phi}<0$.

Claim $\frac{2}{w+b-h}+\frac{\pi'\beta}{1+\pi(h)\beta+\phi}+\frac{1+\pi(h)\beta+\phi}{\pi'\beta(w+b-h)^2}+\ln\left[\frac{\beta(1+r)(w+b)}{1+\pi(h)\beta+\phi}\right]>1$ when w+b is large enough.

The proof to this claim is trivial since $\frac{\pi'\beta}{1+\pi(h)\beta+\phi}$ is bounded and positive; $\frac{2}{w+b-h}$ is positive ; $\frac{1+\pi(h)\beta+\phi}{\pi'\beta(w+b-h)^2}$ is positive. When w+b is large enough, $\ln\left[\frac{\beta(1+r)(w+b)}{1+\pi(h)\beta+\phi}\right]$ becomes large enough such that the LHS>1. Q.E.D.

Proposition 3 is intuitive because as total income increases we can expect health investment increases when income reaches a certain level. When the total income is relatively small, the opportunity cost of investment in health goes to infinity; individuals would allocate the marginal income to consumption, rather than to investment on health. While the motivation of leaving planned bequests decreases, individuals care more about themselves, willing to invest more on health care.

It is worth noting that equation (18) and (21) are different. Equation (18) is the optimal decision of health investment when the consumer is given an annuity contract (A,α). From the consumer's view, the annuity return $(1+\alpha)$ is constant, or exogenously given, not necessarily subject to "the actuarial fairness condition". However, with full information, the annuity firm can contract h, which is endogenized in the annuity return. Comparing equation (18) and (21), we find an extra term in (21), $-\beta(1+r)A\pi'/[\pi(h)C_2]$. Along with equations (16a), (17a) and (3a), it can be simplified to $(-\beta\pi')$, whose absolute value is the discounted marginal rate of survival due to an increase in one unit of health investment.

5.2.2. Private-information: Moral Hazard

In this case, we will assume that annuity firms cannot observe health investment taken by individuals. Individuals can take hidden actions to improve their longevity and thus annuity returns, which is a moral hazard problem. Individuals choose the optimal level of h in response to a contract (A,α) offered by annuity firms.

We use *First Order Approach* (see,e.g., Davies and Kuhn 1992) to analyze the firm's problem. When individuals take hidden actions to invest in health, competitive firms offer an indirect-utility-maximizing actuarially fair contract (A,α), subject to the constraint that individuals choose privately-optimal level of h, which is the first order condition given by (18). Firms do not care individual decisions on savings and bequests because these variables do not affect the firm's profit.

The problem of annuity firms under moral hazard is

$$Max_{A,\alpha} U = \ln(w + b - A - s - h) + \beta\pi(h)\ln[(1+\alpha)A + (1+r)s - b'] + \phi\pi(h)\ln b' + \phi[1 - \pi(h)]\ln[(1+r)s]$$

$$s.t. \quad 1 + r \geq (1+\alpha)\pi(h)$$

where s, b', h given by equations (16) , (17), and (18).

Note that the inequality constraint is non-binding. Similar to the full-information case, we can substitute the inequality with the equality constraint. By "the envelope theorem", the first order condition w.r.t. A is

$$\frac{1}{C_1} = \frac{\beta(1+r)}{C_2}(1-\eta) \tag{23}$$

where $\eta = \frac{\partial\pi}{\partial A}\cdot\frac{A}{\pi} = \frac{\partial h}{\partial A}\cdot\frac{A\pi'}{\pi}$ is the survival elasticity of annuity, which measures the rate of change in survival due to the change in annuity quantity offered by annuity firms. Technically, $\frac{\partial h}{\partial A}$ can be derived from equations (16)-(18) by using "the implicit function theorem". From the annuity firm's point, the largest concern when offering the contract in a moral hazard case is how people would affect their own longevity (by investing on health) in respond to any change in annuity quantity. In a moral hazard case, if $\eta = 0$, the Euler equation is undistorted; if $0 < \eta < 1$, $\frac{C_2}{C_1} < \beta(1+r)$; if $\eta < 0$, $\frac{C_2}{C_1} > \beta(1+r)$.

Proposition 4: *In a pure moral hazard case, a competitive equilibrium is characterized by the decisions of* (A, α, s, h, b')*, such that the altruistic individuals maximize utility by choosing optimal savings s, health investment h, and intentional bequests* b'*, taking the quantity and return of annuities as given; annuity firms offer indirect- utility-maximizing contracts (A,α) subject to the non-negative profit condition and the constraint that individuals choose privately-optimal health investment when given any contract. The solution to* A, s, h *are given by the following system of non-linear equations; the solution to α and* b' *can be explicitly derived from this system.*

$$\frac{1}{w+b-A-s-h} = \left[1-\eta\right]\frac{\beta+\phi}{\frac{A}{\pi(h)}+s} \tag{24}$$

$$\frac{1}{w+b-A-s-h} = \frac{(\beta+\phi)\pi(h)}{\frac{A}{\pi(h)}+s} + \frac{[1-\pi(h)]\phi}{s} \tag{25}$$

$$\frac{1}{w+b-A-s-h} = \pi'\psi + \pi'(\beta+\phi)\ln\left[(1+r)(\frac{A}{\pi(h)}+s)\right] - \pi'\phi\ln\left[(1+r)s\right] \tag{26}$$

where $\eta = \frac{\partial \pi}{\partial A}\cdot\frac{A}{\pi} = \frac{\partial h}{\partial A}\cdot\frac{A\pi'}{\pi}, \psi = \beta\ln\left(\frac{\beta}{\beta+\phi}\right)+\phi\ln\left(\frac{\phi}{\beta+\phi}\right)$.

Equation (24) is the Euler equation under a moral hazard economy, which depends on the survival elasticity of annuity quantity. Equation (25) represents the optimal decision on regular savings, in the same form as in a full-information case. Equation (26) is the optimal health decision with moral hazard. This system of nonlinear equations gives a different solution from that in a full-information case, by recognizing the fact that the RHS of equation (26) is strictly greater than the RHS of equation (21)—the optimal decision on health investment with full-information. Thus, the solution in full-information cannot satisfy the above equation system; the optimal decisions on regular savings, bequests, and health investment are strictly distorted when moral hazard is present!

We can also describe the relation between savings and annuities in a moral hazard case. Equations (23), (16) and (17) give

$$\frac{A}{s} = \frac{\beta\pi(h)}{\phi} - \frac{\eta(\beta+\phi)\pi(h)}{\phi\left[1-\pi(h)\right]} \tag{27}$$

Compared to result (I), an extra term $-\frac{\eta(\beta+\phi)\pi(h)}{\phi\left[1-\pi(h)\right]}$ appears in equation (27).

The relation between intentional and accidental bequests given by equations (23), (16), and (17) is

$$b' = (1+r)s \cdot \frac{1-\eta-\pi(h)}{1-\pi(h)} \tag{28}$$

Compared to result (III), an extra term $\frac{1-\eta-\pi(h)}{1-\pi(h)}$ appears in equation (28). Since $\eta<1$, we have $b'>(1+r)s$, if $\eta<0$; $b'<(1+r)s$, if $0<\eta<1-\pi(h)$; $b'=(1+r)s$, if $\eta=0$. Since both intentional and accidental bequests cannot be non-positive, it is impossible that $1-\pi(h)\leq\eta<1$.

Therefore, we can still have result (I) and result (III) hold in a pure moral hazard case only if $\eta=0$, which means people do not respond to any changes in the annuity quantity given by the contract. However, the optimal decision on health investment reveals that the solution to this case is strictly distorted compared to a full-information case.

A special case is $\phi=0$. To have a better understanding of the first order condition constraint and its role in welfare, we consider a special case of a moral hazard economy when the individual cares only her own consumption without taking bequests into account. With this assumption, we will have the following modifications of our model.

First, non-altruistic individuals will not leave any bequest to offspring, or $b=0$. Second, according to Yaari (1965), the non-altruistic individuals must fully annuitize their savings, which means savings that earn market interest rate should be zero, or $s=0$. The utility of non-altruistic individuals is

$$U=\ln C_1+\beta\pi(h)\ln C_2, \tag{29}$$

where $C_1=w-A-h$, $C_2=(1+\alpha)A$.

For computation simplicity, we use linear survival rate function (see,e.g.,Platoni, S., 2008)

$$\pi(h) = \frac{h}{w} \qquad h \in (0, w)$$

$\pi(h) \in (0,1)$, $\pi \to 1$ as $h \to w$ and $\pi \to 0$ as $h \to 0$. Here, $\pi' = \frac{1}{w}$ is constant.

Given any contract (A, α), consumer's optimal health investment is decided by

$$Max_h U = \ln(w - A - h) + \beta\pi(h)\ln[(1+\alpha)A]$$

F.O.C (h) $$\frac{1}{w-A-h} = \frac{\beta}{w}\ln[(1+\alpha)A] \tag{30}$$

Under full information, health investment is contractible, and the competitive annuity firm offers a utility maximizing contract and designs a level of health investment, subject to the non-negative profit condition.

$$Max_{A,h} U = \ln(w - A - h) + \beta\pi(h)\ln[\frac{(1+r)A}{\pi(h)}]$$

F.O.C (A) $$\frac{1}{w-A-h} = \frac{\beta h}{wA} \tag{31}$$

(h) $$\frac{1}{w-A-h} = \frac{\beta}{w}\ln[\frac{(1+r)Aw}{h}] - \frac{\beta}{w} \tag{32}$$

The solution of (A, h) is implicitly given by equation (31) and (32).

Under a moral hazard economy, health investment cannot be observed, and competitive annuity firms offer utility maximizing contract subject to the consumer's first order condition constraint and non-negative profit condition.

$$Max_A U = \ln(w - A - h) + \beta\pi(h)\ln[\frac{(1+r)A}{\pi(h)}]$$

$$s.t. \quad \frac{1}{w - A - h} = \frac{\beta}{w}\ln\left[\frac{(1+r)A}{\pi(h)}\right] \tag{33}$$

From the constraint, we know that h can be implicitly expressed by A, meaning the consumer chooses a privately-optimal level of health investment when given any contract. By implicit function theorem, we have

$$\frac{\partial h}{\partial A} = -\frac{h[Aw - \beta(w - A - h)^2]}{A[hw + \beta(w - A - h)^2]} \tag{34}$$

The first order condition w.r.t. A is

$$\frac{1}{w - A - h} = \frac{\beta h}{Aw}\left(1 - \frac{\partial h}{\partial A}\cdot\frac{A}{h}\right) \tag{35}$$

where $\frac{\partial h}{\partial A}\cdot\frac{A}{h}$ is the elasticity of annuity purchase to health investment. The solution of (A, h) is implicitly given by equation (33) and (35).

Table 1 shows the numerical results of consumption, annuities, health investment and welfare under a full-information case and under a moral hazard economy. The parameterization is $w = 1000, \beta = 0.3, r = 0.1$.

Table 1. Moral Hazard Equilibrium

	Full-information	Moral Hazard
A	47.1204	0.8329
h	212.0026	29.4530
C1	740.8766	969.7141
U	6.9576	6.9074

This parameterization generates the desired properties of decisions in a full-information case and a moral hazard case: people tend to shrink the health investment and annuity purchases in a moral hazard situation. We provide other parameterization results in appendix. Table 1(a) and 1(b) illustrate the effects of discount factor and the interest rate. As shown in both tables, people tend to increase both annuity purchase and health investment as the value of discount factor increases in a full-information case; however, people in a moral hazard case decrease both choices when the value of discount factor increases. The change of the interest rate does not have significant impact on people's decisions in both cases. Admittedly, the numerical results cannot be extended to a full range of parameter values. In some domain of the parameter space, we may either have negative value of the choice variables, or no solutions.

The numerical results show that in a pure moral hazard case, people significantly reduce health care and annuity purchases, and the welfare level is lower. The reduction in utility in a moral hazard case is equivalent to the utility of a consumer in a full-information case where her income is reduced by 76.9%. This reduction of total income can is the cost of private information!

5.2.3. Private-information: Adverse Selection

We consider the case where individuals are heterogeneous according to the preferences (β, ϕ). The population is partitioned into two distinct groups, L and H, whose relative sizes are fixed. Individuals in group L have low level of preference (β^L, ϕ^L) for future consumption and bequests while individuals in group H have high level of preference (β^H, φ^H), where $0 < \beta^i, \phi^i < 1$, $i = L, H$ and $\beta^L < \beta^H$, $\phi^L < \phi^H$. Such heterogeneity would be reflected in the endogenous survival rate by affecting health investment of people in both groups. People who have high level of health investment would probably have high survival rates and thus low annuity return. However, if they purchase contracts particularly designed for people with low survival rates (at high return), the annuity firms may suffer negative profits. This is an adverse selection problem.

In order to illustrate a pure adverse selection problem, we preclude the case of moral hazard by setting that health investment h^{*i}, $(i = H, L)$ is given by the "health investment rule" as in the full–information case, equation (22). Therefore $h^{*i}, (i = H, L)$ is not a choice variable any more. Besides, we choose the range of (β^i, ϕ^i) such that $\beta^L < \beta^H, \phi^L < \phi^H$ and $h^{*H} > h^{*L}$.Since h^{*i} is fixed, the survival rate $\pi(h^{*i})$ $(i = H, L)$ is fixed and $\pi(h^{*L}) < \pi(h^{*H})$. In the following discussion, we use $\pi^i, (i = H, L)$ to denote the exogenous survival rates of different types of agents.

Now, we introduce "the incentive constraint (IC)". If a type H person purchases the annuity contract designed for a type L person, her indirect utility is given by

$$U^{H(L)}(A^L,\alpha^L)=\ln(w+b-s^{H(L)}-A^L-h^{*H})+\beta^H\pi^H\ln[(1+\alpha^L)A^L+(1+r)s^{H(L)}-b'^{H(L)}]$$
$$+\phi^H\pi^H\ln b'^{H(L)}+\phi^H(1-\pi^H)\ln[(1+r)s^{H(L)}]$$

where $s^{H(L)}(A^L,\alpha^L),b'^{H(L)}(A^L,\alpha^L)\in\arg Max_{s,b'}U(A^L,\alpha^L,s,b')$

Solve $s^{H(L)},b'^{H(L)}$:

$$MaxU_{s,b'}=\ln(w+b-s-A^L-h^{*H})+\beta^H\pi^H\ln[(1+\alpha^L)A^L+(1+r)s-b']+$$
$$\varphi^H\pi^H\ln b'+\varphi^H(1-\pi^H)\ln[(1+r)s]$$

$$(s)\qquad \frac{1}{C_1^{H(L)}}-\frac{\pi^H\beta^H(1+r)}{C_2^{H(L)}}=\frac{\phi^H(1-\pi^H)}{s^{H(L)}} \tag{36}$$

$$(b')\qquad \frac{\beta^H}{C_2^{H(L)}}=\frac{\varphi^H}{b'^{H(L)}} \tag{37}$$

Manipulating (36) and (37), we have

$$\frac{1}{w+b-s^{H(L)}-A^L-h^{*H}}-\frac{\pi^H(\beta^H+\phi^H)}{\dfrac{A^L}{\pi^L}+s^{H(L)}}=\frac{\phi^H(1-\pi^H)}{s^{H(L)}} \tag{36a}$$

$$b'^{H(L)}=\frac{\varphi^H}{\beta^H+\varphi^H}[A^L(1+\alpha^L)+s^{H(L)}] \tag{37a}$$

Apparently from equation (36a) and (37a), $s^{H(L)},b'^{H(L)}$ can be expressed by A^L,α^L.

The incentive constraint (IC) is

$$U^H(A^H,\alpha^H)\geq U^{H(L)}(A^L,\alpha^L) \tag{38}$$

The inequality (38) means that the contract designed for group L people must not be more attractive to members of group H than the contract designed for group H people.

Here we focus on a separating equilibrium where competitive firms offer indirect-utility-maximizing non-negative profit contracts for both groups of people subject to the incentive constraint. It is worth-noting that cross-subsidization among contracts in any given firm is impossible because the firm will withdraw contracts persistently earning negative profits. Especially, since a contract (A^L, α^L) earns non-negative profit from a type L agent only, if a type H agent purchases it, the firm may earn negative profit. Therefore the annuity firm offers an indirect-utility-maximizing contract (A^L, α^L) for a type L agent subject to the incentive constraint and non-negative profit constraint.

$$Max_{A^i, \alpha^i, i=H,L} U^L(A^L, \alpha^L)$$

$$s.t \qquad U^H(A^H, \alpha^H) \geq U^{H(L)}(A^L, \alpha^L)$$

$$1 + r \geq \pi^i (1 + \alpha^i), i = L, H$$

where $s^i, b'^i, i = L, H$ given by equations (16) and (17) with superscript $i, i = L, H$.

Let q be the Lagrangian multiplier; $s^{H(L)}, b'^{H(L)}$ are given by (36) and (37); $h^{*i}, i = L, H$ is fixed from equation (22) with superscript $i, i = L, H$. Still, the inequality is non-binding, which can be substituted with the equality. Applying "the envelope theorem", we have first order conditions for $A^i, i = L, H$

$$(A^L) \qquad \frac{1}{C_1^L} - \frac{\beta^L (1+r)}{C_2^L} = q\{\frac{1}{C_1^{H(L)}} - \frac{\pi^H}{\pi^L} \frac{\beta^H (1+r)}{C_2^{H(L)}}\} \tag{39}$$

$$(A^H) \qquad q[\frac{1}{C_1^H} - \frac{\beta^H(1+r)}{C_2^H}] = 0 \tag{40}$$

Equations (39) gives the optimal annuity quantity purchased by type L people while equation (40) gives the quantity purchased by type H people in an adverse selection economy.

Proposition 5: *In a pure adverse selection case, a competitive equilibrium is characterized by the decisions of* $(A^i, \alpha^i, s^i, h^{*i}, b'^i), i = L, H$ *, such that the altruistic individual i,i=L,H maximizes utility by choosing optimal savings* $s^i, i = L, H$ *and bequests* $b'^i, i = L, H$ *, taking the optimal health investment* $h^{*i}, i = L, H$ *, the quantity and return of annuities as given. If the incentive constraint is non-binding, or* $q = 0$ *, annuity firms offer separate indirect-utility-maximizing zero-profit contracts to members in each group as they would offer in a full-information case. If the incentive constraint is binding, or* $q \neq 0$*, annuity firms offer the same contract to a type H person as they would offer in a full-information case, a different-from- full-information-case contract to a type L person. The solution to* A^L, s^L *and the Lagrangian multiplier q are given by the following system of nonlinear equations; the solution to* α^L *and* b'^L *can be explicitly derived from this system.*

$$\begin{aligned}
&\ln(w + b - s^{H(L)} - A^L - h^{*H}) + \pi^H \psi^H + (\beta^H + \phi^H)\pi^H \ln[(1+r)(\frac{A^L}{\pi^L} + s^{H(L)})] \\
&+ \phi^H(1-\pi^H)\ln[(1+r)s^{H(L)}] = \ln(\frac{w+b}{1+\pi^H\beta^H + \phi^H}) + \beta^H\pi^H \ln[\frac{\beta^H(1+r)(w+b)}{1+\pi^H\beta^H + \phi^H}]
\end{aligned} \tag{41}$$

$$\frac{1}{w+b-s^L-A^L-h^{*L}}-\frac{\beta^L+\phi^L}{\frac{A^L}{\pi^L}+s^L}=q\{\frac{1}{w+b-s^{H(L)}-A^L-h^{*H}}-\frac{\pi^H(\beta^H+\phi^H)}{A^L+\pi^L s^{H(L)}}\} \tag{42}$$

$$\frac{1}{w+b-s^L-A^L-h^{*L}}-\frac{\pi^L(\beta^L+\phi^L)}{\frac{A^L}{\pi^L}+s^L}=\frac{\phi^L(1-\pi^L)}{s^L} \tag{43}$$

where $s^{H(L)}$ can be expressed by A^L, given by (36a).

Equation (41) is from the incentive constraint $U^{H(L)}=U^H$, where we can implicitly solve for A^L. The RHS of (41) $-U^H$ can be easily derived from a full-information case of group H people. Equation (43) can be used to solve for s^L, when the value of A^L is given by (41). Equation (42) can be used to determine the value of q after A^L and s^L are solved.

Note that when the incentive constraint is binding, $q \neq 0$, the Euler equation of a type L person, equation (39), is strictly distorted under an adverse selection problem because of the fact

$$\frac{1}{C_1^{H(L)}}-\frac{\pi^H}{\pi^L}\frac{\beta^H(1+r)}{C_2^{H(L)}} \neq 0$$

Suppose if $\frac{1}{C_1^{H(L)}}-\frac{\pi^H}{\pi^L}\frac{\beta^H(1+r)}{C_2^{H(L)}}=0$, we have a linear relation between A^L and $s^{H(L)}$,

$$s^{H(L)}=\frac{\pi^H(\beta^H+\phi^H)(w+b-h^{H*})}{\pi^L+\pi^H(\beta^H+\phi^H)}-\frac{1+\pi^H(\beta^H+\phi^H)A^L}{\pi^L+\pi^H(\beta^H+\phi^H)}$$

This contradicts to equation (36a), a non-linear relation between A^L and $s^{H(L)}$.

We conclude that the consumption path of a type L agent is strictly distorted in a pure adverse selection economy because the existence of type H agents generates an externality. Due to the existence of type H agents, annuity firms offer a type L agent a contract that if a type H agent purchases it, the type H agent would gain the same utility as she purchases a contract particularly designed for her. The result is that a type H agent is offered a contract she prefers most and without any distortion compared to a first best case, while a type L agent is offered a contract leading to a strict distortion due to the imposition of incentive constraint.

A special case is $\phi^H = \phi^L = 0$ and $h^{*H} = h^{*L} = 0$. To have a better understanding of the adverse selection and its role in welfare, we consider a special case where the individuals are non-altruistic. Besides, since this is a pure adverse selection problem, we eliminate the effect of moral hazard by setting zero health investment. With this assumption, we will have the following modifications of our model.

First, non-altruistic individuals will not leave any bequest to the offspring, or $b = 0$. Second, according to Yaari (1965), the non-altruistic individuals must fully annuitize their savings, which means savings that earn market interest rate should be zero, or $s = 0$. The survival rate π^i of a type i agent $(i = H, L)$ is exogenously given and assume $\pi^H > \pi^L$. Given a survival rate π^i, the return on each unit of annuity of a zero-profit contract is $\frac{1+r}{\pi^i}$, $(i = H, L)$.

With full-information, competitive firms offer a type i agent $(i = H, L)$ utility-maximizing zero-profit contract (A^i, α^i), $(i = H, L)$.

$$Max_{A^i} U^i = \ln C_1^i + \beta^i \pi^i \ln C_2^i$$

where $C_1^i = w - A^i$, $C_2^i = \dfrac{(1+r)A^i}{\pi^i}$, $i = H, L$

The first order condition is

$$\frac{1}{C_1^i} = \frac{\beta^i \pi^i}{A^i}, \quad i = H, L \tag{44}$$

which gives the optimal A^i, C_1^i, C_2^i

$$A^i = \frac{\beta^i \pi^i w}{1+\beta^i \pi^i}, \quad C_1^i = \frac{w}{1+\beta^i \pi^i}, \quad C_2^i = \frac{w\beta^i (1+r)}{1+\beta^i \pi^i}, \quad i = H, L$$

Since $\beta^H > \beta^L$ and $\pi^H > \pi^L$,

$$A^H > A^L, \quad C_1^H < C_1^L$$

and the budget constraint can be written as

$$C_1^i + \frac{C_2^i \pi^i}{1+r} = w. \tag{45}$$

Under an adverse selection problem, annuity firms offer utility-maximizing actuarially fair contract to type L agents subject to the incentive constraint.

$$Max_{A^L, A^H} U^L$$

$$s.t. \quad U^H \geq U^{H(L)}$$

$$L=\ln(w-A^L)+\beta^L\pi^L\ln[\frac{(1+r)A^L}{\pi^L}]+q\{\ln(w-A^H)+\beta^H\pi^H\ln[\frac{(1+r)A^H}{\pi^H}]-$$
$$\ln(w-A^L)-\beta^H\pi^H\ln[\frac{(1+r)A^L}{\pi^L}]\}$$

$$(A^L)\qquad \frac{1}{C_1^L}-\frac{\beta^L\pi^L}{A^L}=q(\frac{1}{C_1^L}-\frac{\beta^H\pi^H}{A^L}) \tag{46}$$

$$(A^H)\qquad q(\frac{1}{C_1^H}-\frac{\beta^H\pi^H}{A^H})=0 \tag{47}$$

q cannot be equal to 1 according to equation (46). If the incentive constraint is binding, or, $q \neq 0$,from equation (46), we have

$$\frac{1}{C_1^L}=\sigma\cdot\frac{\beta^L\pi^L}{A^L} \tag{48}$$

where $\sigma=\frac{1-q\cdot\frac{\beta^H\pi^H}{\beta^L\pi^L}}{1-q}\neq 1$. This means the consumption path of a type L agent is strictly distorted. Hence we can rule out the case where annuity firms offer a type L person the same contract in an adverse selection economy as they would offer in a full-information case.

From equation (48) and the definition of q, we have the following inequalities

$$\sigma>0,\qquad q>0.$$

Hence, we can settle the range of q,

$$0 < q < \frac{\beta^L \pi^L}{\beta^H \pi^H} \quad \text{or} \quad q > 1$$

Claim when $0 < q < \frac{\beta^L \pi^L}{\beta^H \pi^H}$, $\sigma < 1$; when $q > 1$, $\sigma > 1$.

Proof. Since $q > 0$, we have $q\frac{\beta^H \pi^H}{\beta^L \pi^L} > q$, and thus $1 - q\frac{\beta^H \pi^H}{\beta^L \pi^L} < 1 - q$.

When $0 < q < \frac{\beta^L \pi^L}{\beta^H \pi^H}$, we have $1 - q\frac{\beta^H \pi^H}{\beta^L \pi^L} > 0$ and $1 - q > 0$. Therefore,

$$\sigma = \frac{1 - q\frac{\beta^H \pi^H}{\beta^L \pi^L}}{1 - q} < 1.$$

When $q > 1$, we have $1 - q\frac{\beta^H \pi^H}{\beta^L \pi^L} < 0$ and $1 - q < 0$.Therefore,

$q\frac{\beta^H \pi^H}{\beta^L \pi^L} - 1 > q - 1$, and we have

$$\sigma = \frac{1 - q\frac{\beta^H \pi^H}{\beta^L \pi^L}}{1 - q} > 1. \quad \text{Q.E.D.}$$

We can establish a result here that when $0 < q < \frac{\beta^L \pi^L}{\beta^H \pi^H}$, the consumption of a type L agent in the first period is strictly distorted: the agent tends to increase C_1^L and decrease A_1^L; when $q > 1$, consumption of a type L agent in the first period is strictly distorted: the agent tends to decrease C_1^L and increase A_1^L. To have a clear view on this result, we can solve for A^L under an adverse selection economy,

$$A^L = \frac{\sigma\beta^L\pi^L w}{1+\sigma\beta^L\pi^L} \tag{49}$$

If $\sigma < 1$, A^L in moral an adverse selection case is less than A^L in a full-information case, and thus consumption increases. If $\sigma > 1$, A^L in moral an adverse selection case is greater than A^L in a full-information case, and thus consumption decreases. However, whether $0 < q < \frac{\beta^L\pi^L}{\beta^H\pi^H}$ or $q > 1$ depends on the value of parameters.

Table 2 and 3 show a numerical model where values of β^L are varied to illustrate both case. Both tables have the same value of $\beta^H = 0.6$, $\pi^H = 0.9, \pi^L = 0.8, w = 1000$, $r = 0.1$.

Table 2. Adverse Selection Equilibrium, beta(L)=0.3

	Full-information	Adverse Selection
A(H)	350.6493	350.6493
A(L)	193.5485	493.1494
C1(H)	649.3507	649.3507
C1(L)	806.4515	506.8506
U(H)	9.7486	9.7486
U(L)	8.0328	7.7928

q=1.6929

Table 3. Adverse Selection Equilibrium, beta(L)=0.58

	Full-information	Adverse Selection
A(H)	350.6493	350.6493
A(L)	316.9398	224.1904
C1(H)	649.3507	649.3507
C1(L)	683.0602	775.8096
U(H)	9.7486	9.7486
U(L)	9.3464	9.3131

q=0.6972

Both tables show that the consumption plan, annuity purchases and utility of a type H agent is unchanged in an adverse selection economy. It can be seen that the utility of a type L agent is lower in an adverse selection economy. For a type L agent, when her discount rate is not close to that of a type H person (table 1), she tends to increase annuity purchases, and thus consumption falls. This is in line with our analysis when $q > 1$. When the discount rate of a type L agent is close to that of a type H agent (table 2), he tends to decrease annuity purchases and increase consumption in the first period, which gives the same result as our analysis when $0 < q < \frac{\beta^L \pi^L}{\beta^H \pi^H}$. The relation between β^L and

A^L is that when a type L agents is not that impatient (table 2), she would reduce her annuity purchases due to a negative externality imposed by a type H agent. When a type L is relatively quite impatient, the annuity plan provided by the firm particularly to a type L agent enables her to smooth consumption. At this point, our work is different from Eckstein and Peled (1985) in that we have two cases to characterize the distortion of consumption plan of a type L agent in a simple adverse selection economy. Their work, by setting $\beta^H = \beta^L = 1$, ignores the case where consumption plan of a type L person can be distorted downwards.

5.2.4. Private-information : Moral Hazard and Adverse Selection

This case is a combination of the moral hazard and adverse selection. Neither health investment nor individual types can be observed. Individuals choose the optimal level of h in response to a contract (A, α) offered by annuity firms and hide their type information when purchasing annuities.

Before we proceed to the solution, we have some new features in our model when both moral hazard and adverse selection problem present. These features distinguish our work from the previous literature. In the past, most studies mainly focus on either a pure moral hazard problem or a pure adverse selection problem, such as Davies and Kuhn (1992), Eckstein et al (1985), and Eichenbaum and Peled (1987). Studies on a pure moral hazard case with endogenous health investment cannot introduce heterogeneity among individuals within the same generation, while studies on a pure adverse selection economy introduce heterogeneity by giving exogenous survival rate but fail to consider the role of health investment in survival rates.

Few studies take into account the case where both problems appear. Platoni (2008) considers the scenario with both types of private information. By introducing a difference in time preference (impatience), such heterogeneity is transformed into health investment and also into survival rates. He shows that individuals who are less impatient would invest more in health care and annuity firms considering these individuals as high-risk group impose the incentive constraint when offering contracts to another group (low-risk group). However, he fails to discuss the role of bequests in consumers' decisions.

Our model considers the case where individuals are altruistic and both types of private information may present. We want to show that given any contract consumers who care more about future with (β^H, ϕ^H) are actually those who would investment more on health care and thus have high survival rates. When given any annuity contract, the consumer's decisions on savings, bequests and health investment are given by equations (16)-(18). We can simplify this system of nonlinear equations to

$$\frac{1}{w+b-A-h-s} = \frac{(\beta+\phi)(1+r)\pi(h)}{(1+\alpha)A+(1+r)s} + \frac{[1-\pi(h)]\phi}{s} \tag{16a}$$

$$\begin{aligned}\frac{1}{w+b-A-h-s} &= \pi'\left[\beta \ln\frac{\beta}{\beta+\phi} + \phi \ln\frac{\phi}{\beta+\phi}\right] + \\ &\pi'(\beta+\phi)\ln[(1+\alpha)A+(1+r)s] - \pi'\phi\ln[(1+r)s]\end{aligned} \tag{18a}$$

Since the above system of nonlinear equations cannot be solved explicitly, we use will numerical results presented in Table 4 in Appendix. Table 4 shows that at some given contracts, people with (β^H, ϕ^H) have significantly high level of health investment than those with (β^L, ϕ^L). This result is consistent with those in literature. Intuitively, people who value future more than others would invest more in health for better chances of survival. However, we need to admit the strong restrictions on parameters when we conduct the simulation: the results are based on particularly selected parameters. In some domain of the parameter space, the solution is negative for some choice variables, or no solutions. The numerical results cannot be extended to the whole range of the parameters, but it provides a particular perspective that is usually expected to happen.

Individuals characterized by (β^H, ϕ^H) are high-risk group for the annuity firm because if they purchase contracts designed for individuals with (β^L, ϕ^L) --low-risk group-- the firm may suffer negative profit. The incentive constraint must be imposed on them in the maximization problem.

The contract offered to individuals with (β^H, ϕ^H) when both problems present should be the same as offered under a pure moral hazard case, while the contract offered to individuals with (β^L, ϕ^L) should be further imposed by the incentive constraint. Different from the pure adverse selection case, the mixed problem allows consumers to choose optimal health investment when given a contract (A, α). That is, $h^i, i = L, H$ is not fixed any more. We have some modifications for the utility of a type *H* person purchasing a contract that designed for a type *L* person.

$$U^{H(L)}(A^L, \alpha^L) = \ln(w + b - s^{H(L)} - A^L - h^{H(L)}) + \beta^H \pi(h^{H(L)}) \ln[(1+\alpha^L)A^L + (1+r)s^{H(L)} - b'^{H(L)}] + \varphi^H \pi(h^{H(L)}) \ln b'^{H(L)} + \varphi^H (1 - \pi(h^{H(L)})) \ln[(1+r)s^{H(L)}]$$

where $s^{H(L)}(A^L,\alpha^L), b'^{H(L)}(A^L,\alpha^L), h^{H(L)}(A^L,\alpha^L) \in \arg Max_{s,b',h} U(A^L,\alpha^L s, b', h)$ .The first order conditions are

$$(s) \quad \frac{1}{w+b-s^{H(L)}-A^L-h^{H(L)}} - \frac{\pi(h^{H(L)})\beta^H(1+r)}{(1+\alpha^L)A^L+(1+r)s^{H(L)}-b'^{H(L)}} = \frac{\phi^H[1-\pi(h^{H(L)})]}{s^{H(L)}} \quad (50)$$

$$(b') \quad \frac{\pi(h^{H(L)})\beta^H}{(1+\alpha^L)A^L+(1+r)s^{H(L)}-b'^{H(L)}} = \frac{\phi^H\pi(h^{H(L)})}{b'^{H(L)}} \quad (51)$$

$$(h) \quad \begin{aligned} &\frac{1}{w+b-s^{H(L)}-A^L-h^{H(L)}} = \pi'(h^{H(L)})\beta^H \ln[(1+\alpha^L)A^L+(1+r)s^{H(L)}-b'^{H(L)}] \\ &+\varphi^H\pi'(h^{H(L)})\ln b'^{H(L)} - \varphi^H\pi'(h^{H(L)})\ln[(1+r)s^{H(L)}] \end{aligned} \quad (52)$$

Equation (52) is new since in a pure moral hazard case, $h^i, i=L,H$ is fixed while in this case we need to relax this assumption. $s^{H(L)}, b'^{H(L)}, h^{H(L)}$ can be expressed in terms of (A^L,α^L) in that type H individuals optimize decisions on savings, bequest and health investment when they choose to purchase a contract designed for type L people.

The incentive constraint (IC) is

$$U^H(A^H,\alpha^H) \geq U^{H(L)}(A^L,\alpha^L)$$

The contract designed for group L people must not be more attractive to members of group H than the contract designed for group H people.

Competitive firms offer the indirect-utility-maximizing contract to a type L individuals subject to the first order constraints of $h^i, i=L,H$, the incentive constraint and non-negative profit condition.

$$Max_{A^i,\alpha^i,i=H,L} U^L(A^L,\alpha^L)$$

$$s.t \quad 1+r \geq \pi(h^i)(1+\alpha^i) \quad , \ i=L,H$$

$$U^H(A^H,\alpha^H) \geq U^{H(L)}(A^L,\alpha^L) \qquad (q)$$

where $s^i, h^i, b'^i, i=L,H$ given by

$$\frac{1}{w+b-A^i-s^i-h^i} - \frac{\pi(h^i)(\beta^i+\phi^i)}{\frac{A^i}{\pi(h^i)}+s^i} = \frac{[1-\pi(h^i)]\phi^i}{s^i} \quad , \ i=L,H$$

$$\frac{1}{w^i+b^i-A^i-s^i-h^i} = \pi'^i\psi^i + \pi'^i(\beta^i+\phi^i)\ln[(1+r)(\frac{A^i}{\pi(h^i)}+s^i)] - \pi'^i\phi^i\ln[(1+r)s^i] \quad , \ i=L,H$$

$$b'^i = \frac{\phi^i(1+r)}{\beta^i+\phi^i}[\frac{A^i}{\pi^i}+s^i], i=L,H$$

The first order conditions for $A^i, i=L,H$ are

$$(A^L) \quad \frac{1}{C_1^L} - \frac{\beta^L(1+r)}{C_2^L}(1-\eta^L) = q[\frac{1}{C_1^{H(L)}} - \frac{\beta^H(1+r)}{C_2^{H(L)}}\cdot\frac{\pi(h^{H(L)})}{\pi(h^L)}(1-\eta^L)] \qquad (53)$$

$$(A^H) \quad q[\frac{1}{C_1^H} - \frac{\beta^H(1+r)}{C_2^H}(1-\eta^H)] = 0 \qquad (54)$$

where $\eta^i = \frac{\partial\pi^i}{\partial A^i}\cdot\frac{A^i}{\pi^i}, i=L,H$ is the survival elasticity due to the change in annuity quantity. If the constraint is nonbinding, or, $q=0$, it is a pure moral hazard case for both types of individuals.

The results show that if the incentive constraint is binding, or $q \neq 0$, a contract provided to a type *H* agent would be the same as the firm offers under a pure adverse selection problem. And a type *H* person would choose the same optimal decisions as she would choose in a pure adverse selection problem.

However, if $q \neq 0$, for a type *L* agent, his contract is characterized by the features from both pure moral hazard and adverse selection problem. The existence of type *H* agents put an externality on type *L* agents. A special case where agents do not have bequests motives has been investigated by Platoni (2008) and the similar results are derived in that paper.

6. Conclusion

When agents face uncertain lifetime, concerns about offspring lead to bequest motivation. The economy we describe here are full of agents valuing both intentional and accidental bequests. We have explored the role of intentional and accidental bequests in a maximum-two-period-living consumer's optimal resource allocation decisions. We show that under certain circumstances individuals tend to leave the same amount of accidental and intentional bequests.

We analyze the role of information in annuity contract design. Four cases are discussed. In a full information case, agents leave the same amount of accidental and intentional bequests, and health investment is contractible so that from annuity firms' view, the contract earns non-negative profit.

In a pure moral hazard case, we recognize a distortion in consumption due to private information. The annuity firms in a pure moral hazard economy would prevent the potential risk in negative profit and take consumer's private information into account. Such preventions would lower welfare level of consumers than under a full information case, which can be better understood in a special case. In a pure adverse selection case, we see the effect of heterogeneity in individuals' decisions and contract design. The existence of a high risk group (from firms' view) imposes an externality on a low risk group whose consumption and other decisions are thus distorted. In the presence of both moral hazard and adverse selection problem, a separating equilibrium implies that both groups have the same characteristics as under a moral hazard case, and moreover, decisions of the low risk group are further distorted due to the externality imposed by the high risk group.

We use techniques such as first order condition constraint and incentive-compatibility constraint fully characterize solutions in each case, casting a shadow on future studies on annuity markets. We also provide simplified cases and numerical results to gain a better understanding of the problem we elaborate.

Finally, further research may extend the exploration in several aspects. First, an initial pooling equilibrium may be considered under private information, which needs to consider both Rothschild/Stiglitz equilibrium and Wilson equilibrium. Second, it is tempting to extend the model to a dynamic framework by introducing an overlapping generation model. However, researchers working on that would be more careful to handle the existence of intentional and accidental bequests. Such heterogeneity in bequests exists in both inter-generation and intra-generation. Third, a social security program can be introduced to such a model in the aim of improving welfare.

REFERENCE

Abel, A. 1985. "Precautionary Savings and Accidental Bequests", American Economic Review 75, 777-791.

Altonji, J.G., F. Hayashi, and Kotlikoff, L.J. 1997. "Parental Altruism and Inter Vivos Transfers: Theory and Evidence", Journal of Political Economy 105, 1121-1166.

Auerbach, A.J. and Kotlikoff, L.J. 1992. "The Impact of the Demographic Transition on Capital Formation", Scandinavian Journal of Economics 94, 281-295.

Davies,J., and Kuhn, P.1992. “Social Security, Longevity, and Moral Hazard”, Journal of Public Economics 49, 91-106.

Eckstein,Z.,Eichenbaum,M., and Peled,D.1985.“Uncertain Lifetime and the Welfare Enhancing Properties of Annuity Markets and Social Security”, Journal of Public Economics 26, 303-326.

Ehrlich, I. and Lui, F.T.1991. "Intergenerational Trade, Longevity, and Economic Growth", Journal of Political Economy 99, 1029-1060.

Eichenbaum,M., and Peled, D.1987. “Capital Accumulation and Annuities in an Adverse Selection Economy”, The Journal of Political Economy 95, 334-354.

Huggett, M. 1996. "Wealth Distribution in Life-Cycle Economics", Journal of Monetary Economics 38, 469-494.

Kotlikoff, L.J. and Summers, L.H. 1981. "The Role of Intergenerational Transfers in Aggregate Capital Accumulation", Journal of Political Economy 89, 706-732.

Pauly, M. 1974. “Overinsurance and Public Provision of Insurance: The Role of Moral Hazard and Adverse Selection”, The Quarterly Journal of Economics 88, 44-62.

Pecchenino, R., and Pollard, P.1997. “The Effect of Annuities, Bequests, and Aging in an Overlapping Generations Model of Endogenous Growth”, The Economic Journal 107, 26-46.

Platoni,S. 2008. “Asymmetric information and annuities”, Unpublished Manuscript.

Yaari,M.1965. “Uncertain Lifetime, Lifetime Insurance, and the Theory of the Consumer”, The Review of Economic Studies 32,137-150.

Zhang,J.,Tang,K.K. 2008. “Morbidity, Longevity and Annuitization with Uninsurable Medical Expenses”, Unpublished Manuscript.

Zhang, J.,Zhang, J and Lee,R. 2003. “Rising Longevity, Education, Savings, and Growth”, Journal of Development Economics 70, 83-101.

Zhang, J.,Zhang, J and Lee,R. 2001. "Mortality Decline and Long-Run Economic Growth", Journal of Public Economics 80, 485-507

APPENDIX

Table 1(a). Moral Hazard Equilibrium
r=0.1

	Full-info	MH	Beta
A	47.1204	0.8329	0.3
h	212.0026	29.4530	0.3
A	101.0104	0.4639	0.5
h	454.4633	6.8111	0.5
A	124.1062	0.01451	0.7
h	558.3751	3.8060	0.7
A	136.9371	0.0077	0.9
h	616.1038	2.7633	0.9

Table 1(b). Moral Hazard Equilibrium
r=0.01

	Full-info	MH	Beta
A	45.5772	1.0163	0.3
h	201.8812	32.6085	0.3
A	101.0188	0.5516	0.5
h	447.4562	7.4275	0.5
A	124.7795	0.1723	0.7
h	552.7026	4.1464	0.7
A	137.9799	0.0091	0.9
h	611.1723	3.0097	0.9

Table4 Decisions on health investment

A	beta	phi	h	s
50	0.6	0.4	646.4129	298.8366
	0.3	0.2	169.6992	233.6481
100	0.6	0.4	651.5750	259.9704
	0.3	0.2	190.3282	215.4827
150	0.6	0.4	652.9402	225.4737
	0.3	0.2	201.4364	199.3515
200	0.6	0.4	649.0240	196.3427
	0.3	0.2	203.9505	185.7625
250	0.6	0.4	638.6065	173.1246
	0.3	0.2	198.6732	174.7636
300	0.6	0.4	621.2344	155.6338
	0.3	0.2	186.4960	166.1393
350	0.6	0.4	597.3384	143.0539
	0.3	0.2	168.3739	159.5567
400	0.6	0.4	567.9011	134.3182
	0.3	0.2	145.2391	154.6621
450	0.6	0.4	534.0343	128.4354
	0.3	0.2	117.9351	151.1330
500	0.6	0.4	496.7263	124.6244
	0.3	0.2	87.18505	148.6978

Note: survival rate function is linear $\pi(h) = h / w$. The parameterization is $w = 1500, b = 0$, $r = 0.1, \alpha = 0.2$.